Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name : Mathematics-I

Subject Code : 4SC01MTC1 **Branch : B. Sc. (All)**

Semester : 1 **Date : 4 / 12 / 2015** **Time : 10:30 To 1:30** **Marks :70**

Instructions:

1. Use of Programmable calculator & any other electronic instrument is prohibited.
2. Instructions written on main answer book are strictly to be obeyed.
3. Draw neat diagrams and figures (if necessary) at right places.
4. Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Obtain the cartesian co-ordinates for the polar co-ordinates $\left(1, \frac{\pi}{2}\right)$.

b) Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z - 11 = 0$.

c) If $y = \sin x \cos x$, then find y_n.

d) If $f(x) = \sin x, x \in [0, \pi]$, then find the value of c using Rolle's theorem.

e) Write down the expansion of $\sin x$ in terms of x.

f) If $\lim_{x \to \frac{\pi}{2}} \frac{(\cos x)^2}{a - \sin x} = 1$ then find the value of a.

g) Find the order of differential equation $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$.

h) Find the general solution of $y = xp - p^2 + \log p$.

i) Check whether the differential equation $(\cos y + y\cos x)dx + (\sin x - x\sin y)dy = 0$ is exact or not.

j) Define: trace of a square matrix.

k) Find determinant of $A = \begin{bmatrix} \sin x & -\sec x \\ \cos x & -cosec\, x \end{bmatrix}$.

l) Check whether the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is in row-echelon form or not.

m) State Cayley-Hamilton theorem.

n) Find the eigen values of $A = \begin{bmatrix} 100 & 100 \\ 0 & -100 \end{bmatrix}$.

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Find the inverse of a given matrix if possible,

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}.$$ **[6]**

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	b)	Solve: $\frac{dy}{dx} + x\sin 2y = x^3(\cos y)^2$.	[6]
	c)	Evaluate: $\lim_{x\to a} \frac{x\cos x - \log(1+x)}{x^2}$.	[2]
Q-3		**Attempt all questions**	(14)
	a)	For $x > 0$, show that $\frac{x}{1+x^2} < \tan^{-1} x < x$.	[6]
	b)	Derive the n[th] derivative of $e^{ax}\sin(bx+c)$.	[6]
	c)	If $(2, \frac{\pi}{4}, \frac{\pi}{6})$ are spherical co-ordinates for a point then find its cartesian co-ordinates.	[2]
Q-4		**Attempt all questions**	(14)
	a)	State and prove Leibnitz's theorem.	[7]
	b)	Test for consistency and solve: $5x + 3y + 7z = 4, 3x + 26y + 2z = 9, 7x + 2y + 10z = 5$.	[7]
Q-5		**Attempt all questions**	(14)
	a)	State and prove Cauchy's mean value theorem.	[7]
	b)	State necessary condition for differential equation to be exact and hence find the solution of $(x^2 - ay)dx + (y^2 - ax)dy = 0$.	[7]
Q-6		**Attempt all questions**	(14)
	a)	If cylindrical co-ordinates of the point C are $C\left(2, \frac{7\pi}{6}, 2\sqrt{3}\right)$ then obtain its cartesian and spherical co-ordinates.	[6]
	b)	Find the n[th] derivative of $\frac{x}{(x-1)(2x+3)}$, $x \neq 1, x \neq -\frac{3}{2}$.	[6]
	c)	If $AB = BA$ and $S^2 = B$ then prove that $(A^{-1}SA)^2 = B$.	[2]
Q-7		**Attempt all questions**	(14)
	a)	Find approximate value of $\log 73.55$ correct up to six decimal points, where $\log_{10} 73 = 1.863323, \log_e 10 = 0.43429$.	[6]
	b)	Solve the following system by Gauss-Jordan elimination method $x + y + 2z = 8, -x - 2y + 3z = 1, 3x - 7y + 4z = 10$.	[6]
	c)	Define: skew-Hermitian matrix and give one example of it.	[2]
Q-8		**Attempt all questions**	(14)
	a)	Find the eigen values and eigen vectors of the following matrix, $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$.	[6]
	b)	If $y = (\sin^{-1} x)^2$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$. Hence find $(y_n)_0$.	[6]
	c)	Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point $(1, 2, 3)$.	[2]